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OINT COMMITTEE ON REINFORCED CONCRETE

SECOND REPORT

LONDON
THE ROYAL INSTITUTE OF BRITISH ARCHITECTS
9 CONDUIT STREET, REGENT STREET, W.
1911

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THE JOINT COMMITTEE ON REINFORCED CONCRETE:

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Vice-Chairman: Professor W. C. Unwin, F.R.S., Hon. A.R.I.B.A., President Institution Civil Engineers.

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KEY TO THE NOTATION.

The notation is built up on the principle of an index.

The significant words in any term are abbreviated down to their initial letter, and there are no exceptions.

Capital letters indicate moments, areas, volumes, total forces, total loads, ratios, and constants, &c.

Small letters indicate intensity of forces, intensity of loads, and intensity of stresses, lineal dimensions (lengths, distances, &c.), ratios, and constants, &c.

Dashed letters indicate ratios, such as a, c, n, &c., where the a, c, and n indicate the numerators in the respective ratios. The dash itself is mnemonic and is an abbreviation of that longer dash which indicates division or ratio.

Subscript letters are only used where one letter is insufficient; and the subscript letters themselves are the initial or distinctive letters of the qualifying words.

Greek letters indicate ratios and constants. They are sparingly used and are subject to the "initial letter" principle.

The symbols below are arranged in alphabetical order for facility of reference.

STANDARD NOTATION.

(In pillars) A = the effective *area* of the pillar (see definition on page 15).
A_E = *Area equivalent* to some given area or *area* of an *equivalent section* or equivalent area.
A_s = cross-sectional *area* of a vertical or diagonal *shear* member, or group of shear members, in the length p, where p = pitch of stirrups.
A_t = *Area* of *tensile* reinforcement (in square inches).
(In pillars) A_V = *Area* of *vertical* or longitudinal reinforcement in square inches.
a = *arm* of the resisting moment or lever *arm* (in inches).
$a_{,}$ = *arm* ratio $= a/d \therefore a_{,} d = a$.
B = *Bending* moment of the external loads and reactions (in pound inches).
$B_1\ B_2$ = *Bending* moments at consecutive cross sections.
Generally b = *breadth*.
(In tee beams) b = *breadth* of flange of beam (in inches).
b_r = *breadth* of *rib* of $\top$ beam (in inches).
$C_1\ C_2\ C_3$ = a series of *constants*.
(In beams) c = *compressive* stress on the compressed edge of the concrete (in pounds per square inch).
(In pillars) c = working *compressive* stress on the concrete of the hooped core.
c_p = the working *compressive* stress on a *prism* of concrete (not hooped) or the working *compressive* stress of *plain* concrete.
c_u = *compressive* stress on concrete at the *underside* of the slab (in tee beams).
$c_{,} = c/t$ = the ratio of c to t.
In circular sections
generally d = *diameter*.
In rectangular sections
generally d = *depth*.
(In pillars) d = the *diameter* of the hooped core in inches.
(In beams) d = effective *depth* of the beam (in inches).
(In beams) d_c = *depth* or *distance* of the *centre* of compression from the compressed edge.
d_n = *deflection*.
d_s = total *depth* of the *slab* (in inches).
(In pillars) d_v = *distance* between the centres of *vertical* bars measured perpendicular to the neutral axis.
E_c = *Elastic* modulus of *concrete* (in pounds per square inch).
E_s = *Elastic* modulus of *steel* (in pounds per square inch).
e = *eccentricity* of the load measured from the centre of the pillar (in inches).
(In beams) f = extreme *fibre* stress, *i.e.* stress at the extreme "fibre" of any member under transverse load.
(In pillars) f = a *form* factor or constant which will vary according to whether the hooping is curvilinear or rectilinear, &c.
I = *Inertia* moment of a member.
I_c = *Inertia* moment of *concrete* only.
I_s = *Inertia* moment of *steel* only.
I_{XX} = *Inertia* moment on axis XX when necessary.
I_{YY} = *Inertia* moment on axis YY when necessary.
l = *length* of a pillar or effective length of span of beam or slab.
m = *modular* ratio $= E_s/E_c$.
N = a *numerical* coefficient.
n = *neutral* axis depth, *i.e.* depth of *neutral* axis from the extreme compressed edge (in inches).
$n_{,} = n/d$ = the *neutral* axis ratio $\therefore n_{,} d = n$.
$N_2\ N_3\ N_4$ = a series of numerical coefficients.
P = total safe *pressure*.
(In pillars) p = the *pitch* of the laterals in inches (*i.e.* the axial spacing of the laterals).
(In shear formulæ) p = *pitch* of distance apart (centre to centre) of the shear members or groups of shear members (measured horizontally).
π = *peripheral* ratio or the ratio of the circumference of a circle to its diameter.
R_c = *Compressive Resistance* moment = *Resistance* moment of the beam in terms of the *compressive* stress (in pound inch units).
R_t = *Tensile Resistance* moment or Resistance moment in terms of the *tensile* stress (in pound inches).
(In beams) $r = A_t/bd$ = *ratio* of area of tensile reinforcement to the area bd.
(In pillars) $r = V_h/V$ = the *ratio* of volumes, *i.e.* the ratio of the volume of *helical* or *horizontal* reinforcement to the volume of hooped core.
(In beams) S = the total *shear* in pounds at a vertical section.
S_m = the *section modulus*.
(In pillars) s = *Spacing* factor or constant which will vary with the pitch of the laterals.
(In beams) s = intensity of the *shearing* stress on concrete in pounds per square inch.
s_s = *shearing* stress on the *steel* (in units of force per unit of area).
(In beams) $s_{,} = d_s/d$ = the *slab* depth ratio.
T = Total *tension* in the steel (in pounds).
$T_1\ T_2$ = Total *tensile* forces at consecutive cross sections.
t = *tensile* stress on the steel (in pounds per square inch).
U = Total *ultimate* breaking load on any member. [Compare W = *Working* load.]
u = intensity of *ultimate* crushing resistance of plain concrete per unit of area or ultimate compressive stress on prisms of concrete not hooped.
(In pillars) V = *Volume* of hooped core in cubic inches.
(In pillars) V_h = *Volume* of *hooping* reinforcement in cubic inches.
W = total *working* load or *weight* on any member.
(In pillars) W_F = the *working factor* $= c_p/u$ = the reciprocal of the safety factor.
w = *weight* or load per unit of length of span.

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INTRODUCTORY NOTE.

The First Report of this Committee appeared in 1907. Since that date the use and knowledge of Reinforced Concrete for Architectural and Engineering Constructions have steadily increased. It therefore appeared desirable that it should be reconsidered in the light of further experience, and this Second Report is the result of the Committee's labours.

The section on Materials has been modified in certain details.

The section on Methods of Calculation has been re-cast in form, and the standard notation (proposed by the Concrete Institute) has been adopted.

The sub-section on Columns has been revised and the formulæ proposed have been re-cast so as to include the cases in which the lateral or helical binding is a material factor in the strength.

The suggestions which have been made from time to time by Institutes and individuals, for which the Committee's thanks are due, have been of much value and have been fully considered.

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SECOND REPORT.

PREFATORY NOTE.

1. Reinforced concrete is used so much in building and engineering construction that a general agreement on the essential requirements of good work is desirable. The proposals which follow are intended to embody these essentials, and to apply generally to all systems of reinforcement.

Good workmanship and materials are essential in reinforced concrete. With these and good design structures of this kind appear to be trustworthy. It is essential that the workmen employed should be skilled in this class of construction. Very careful superintendence is required during the execution of the work in regard to—

(*a*) The quality, testing, and mixing of the materials.

(*b*) The sizes and positions of the reinforcements.

(*c*) The construction and removal of centering.

(*d*) The laying of the material in place and the thorough punning of the concrete to ensure solidity and freedom from voids.

If the metal skeleton be properly coated with cement, and the concrete be solid and free from voids, there is no reason to fear decay of the reinforcement in concrete of suitable aggregate and made with clean fresh water.

2. The By-laws regulating building in this country require external walls to be in brick, or stone, or concrete of certain specified thicknesses. In some places it is in the power of the local authorities to permit a reduced thickness of concrete when it is strengthened by metal; in other districts no such power has been retained. We are of opinion that all By-laws should be so altered as to expressly include reinforced concrete amongst the recognised forms of construction.

A section should be added to the By-laws declaring that when it is desired to erect buildings in reinforced concrete complete drawings showing all details of construction and the sizes and positions of reinforcing bars, a specification of the materials to be used and proportions of the concrete, and the necessary calculations of strength based on the rules contained in this Report, signed by the person or persons responsible for the design and execution of the work, shall be lodged with the local authority.

3. Fire Resistance.—(*a*) Floors, walls, and other constructions in steel and concrete formed of incombustible materials prevent the spread of fire in varying degrees according to the composition of the concrete, the thickness of the parts, and the amount of cover given to the metal.

(*b*) Experiment and actual experience of fires show that concrete in which limestone is used for the aggregate is disintegrated, crumbles and loses coherence when subjected to very fierce fires, and that concretes of gravel or sandstones also suffer, but in a rather less degree.* The metal reinforcement in such cases generally retains the mass in position, but the strength of the part is so much diminished that it must be renewed. Concrete in which coke-breeze, cinders, or slag forms the aggregate is only superficially injured, does not lose its strength, and in general may be repaired. Concrete of broken brick suffers more than cinder concrete and less than gravel or stone concrete.

(*c*) The material to be used in any given case should be governed by the amount of fire resistance required as well as by the cheapness of, or the facility of procuring, the aggregate.

(*d*) Rigidly attached web members, loose stirrups, bent-up rods, or similar means of connecting the metal in the lower or tension sides of beams or floor slabs (which sides suffer most injury in case of fire) with the upper or compression sides of beams or slabs not usually injured are very desirable.

(*e*) In all ordinary cases a cover of ½ inch on slabs and 1 inch on beams is sufficient. It is undesirable to make the covering thicker. All angles should be rounded or splayed to prevent spalling off under heat.

(*f*) More perfect protection to the structure is required under very high temperature, and in the most severe conditions it is desirable to cover the concrete structure with fire-resisting plastering which may be easily renewed. Columns may be covered with coke-breeze concrete, terra-cotta, or other fire-resisting facing.

MATERIALS.

4. *Cement.*—Only Portland cement complying with the requirements of the specification adopted by the British Engineering Standards Committee should be employed; in general the slow-setting quality should be used. Every lot of cement delivered should be tested, and in addition the tests for soundness and time of setting, which can be made without expensive apparatus, should be applied frequently during construction. The cement should be delivered on the work in bags or barrels bearing the maker's name and the weight of the cement contained.

5. *Sand.*—The sand should be composed of hard grains of various sizes up to particles which will pass a quarter-inch square mesh, but of which at least 75 per cent. should pass ⅛-inch square mesh. Fine sand alone is not so suitable, but the finer the sand the greater is the quantity of cement required for equal strength of mortar. It should be clean and free from ligneous, organic, or earthy matter. The value of a sand cannot always be judged from its appearance, and tests of the mortar prepared with the cement and the sand proposed should always be made. Washing sand does not always improve it, as the finer particles which may be of value to the compactness and solidity of the mortar are carried away in the process.

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* The smaller the aggregate the less the injury.

6. *Aggregate.*—The aggregate, consisting of gravel, hard stone, or other suitable material,* should be clean and preferably angular, varied in size as much as possible between the limits of size allowed for the work. In all cases material which passes a sieve of a quarter-inch square mesh should be reckoned as sand. The maximum allowable size is usually ¾ inch. The maximum limit must always be such that the aggregate can pass between the reinforcing bars and between these and the centering. The sand should be separated from the gravel or broken stone by screening before the materials are measured.

7. *Proportions of the Concrete.*—In all cases the proportions of the cement, sand, and aggregate should be separately specified in volumes. The amount of cement added to the aggregate should be determined on the work by weight. The weight of a cubic foot of cement for the purpose of proportioning the amount of cement to be added may be taken at 90 lbs. As the strength and durability of reinforced concrete structures depend mostly on the concrete being properly proportioned, it is desirable that in all important cases tests should be made as described herein with the actual materials that will be used in the work before the detailed designs for the work are prepared.

In no case should less dry cement be added to the sand when dry than will suffice to fill its interstices, but subject to that the proportions of the sand and cement should be settled with reference to the strength required, and the volume of mortar produced by the admixture of sand and cement in the proportions arranged should be ascertained.†

The interstices in the aggregate should be measured and at least sufficient mortar allowed to each volume of aggregate to fill the interstices and leave at least 10 per cent. surplus.

For ordinary work a proportion of one part cement to two parts sand will be found to give a strong, practically watertight mortar, but where special watertightness or strength is required the proportion of cement must be increased.

8. *Metal.*—The metal used should be steel having the following qualities:

(*a*) An ultimate strength of not less than 60,000 lbs. per square inch.

(*b*) A yield point of not less than 32,000 lbs. per square inch.

(*c*) It must stand bending cold 180° to a diameter of the thickness of pieces tested without fracture on outside of bent portion.

(*d*) In the case of round bars the elongation should not be less than 22 per cent., measured on a gauge-length of eight diameters. In the case of bars over one inch in diameter the elongation may be measured on a gauge-length of four diameters, and should then be not less than 27 per cent.

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* Coke-breeze, pan breeze or boiler-ashes ought not to be used for reinforced concrete. It is advisable not to use clinker or slag, unless the material is selected with great care.

† For convenience on small works the following figures may be taken as a guide, and are probably approximately correct for medium siliceous sand:—

Parts Cement		Parts Sand		Parts Mortar	Parts Cement		Parts Sand		Parts Mortar
1	+	½	=	1·20	1	+	2	=	2·35
1	+	1	=	1·50	1	+	2½	=	2·70
1	+	1½	=	1·90	1	+	3	=	3·00

For other sectional material the tensile and elongation tests should be those prescribed in the British Standard Specification for Structural Steel. If hard or special steel is used, it must be on the architect's or engineer's responsibility and to his specification.

Before use in the work the metal must be clean and free from scale or loose rust. It should not be oiled, tarred, or painted.

Welding should in general be forbidden; if it is found necessary, it should be at points where the metal is least stressed, and it should never be allowed without the special sanction of the architect or engineer responsible for the design.

The reinforcement ought to be placed and kept exactly in the positions marked on the drawings, and, apart from any consideration of fire resistance, ought not to be nearer the surface of the concrete at any point than 1 inch in beams and pillars and ½ inch in floor slabs or other thin structures.

9. *Mixing : General.*—In all cases the concrete should be mixed in small batches and in accurate proportions, and should be laid as rapidly as possible. No concrete which has begun to set should be used.

Hand-mixing.—When the materials are mixed by hand they are to be turned over dry and thoroughly mixed on a clean platform until the colour of the cement is uniformly distributed over the aggregate.

Machine-mixing.—Whenever practicable the concrete should be mixed by machinery.

10. *Laying.*—The thickness of loose concrete that is to be punned should not exceed three inches before punning, especially in the vicinity of the reinforcing metal. Special care is to be taken to ensure perfect contact between the concrete and the reinforcement, and the punning to be continued till the concrete is thoroughly consolidated. Each section of concreting should be as far as possible completed in one operation *; when this is impracticable, and work has to be recommenced on a recently laid surface, it is necessary to wet the surface ; and where it has hardened it must be hacked off, swept clean, and covered with a layer of cement mortar ½ inch thick, composed of equal parts of cement and sand. Work should not be carried on when the temperature is below 34° Fahr. The concrete when laid should be protected from the action of frost, and shielded against too rapid drying from exposure to the sun's rays or winds, and kept well wetted. All shaking and jarring must be avoided after setting has begun. The efficiency of the structure depends chiefly on the care with which the laying is done.

Water.—The amount of water to be added depends on the temperature at the time of mixing, the materials, and the state of these, and other factors, and no recommendation has therefore been made. Sea-water should not be used.

11. *Centering or Casing.*—The centering must be of such dimensions, and so constructed, as to remain rigid and unyielding during the laying and punning of the concrete. It must be so arranged as to permit of easing and removal without jarring the concrete. Provision should be made wherever

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* In particular the full thickness of floor slab should be laid in one operation.

practicable for splaying or rounding the angles of the concrete. Timber when used for centering may be advantageously limewashed before the concrete is deposited.

12. *Striking of Centres.*—The time during which the centres should remain up depends on various circumstances, such as the dimensions or thickness of the parts of the work, the amount of water used in mixing, the state of the weather during laying and setting, &c., and must be left to the judgment of the person responsible for the work. The casing for columns, for the sides of beams, and for the soffits of floor slabs not more than 4 feet span must not be removed under eight days; soffits of beams and of floors of greater span should remain up for at least fourteen days, and for large span arches for at least twenty-eight days. The centering of floors in buildings which are not loaded for some time after the removal of same may be removed in a short time; the centering for structures which are to be used as soon as completed must remain in place much longer. If frost occurs during setting, the time should be increased by the duration of the frost.

13. *Testing.*—Before the detailed designs for an important work are prepared, and during the execution of such a work, test pieces of concrete should be made from the cement, sand, and aggregate to be used in the work, mixed in the proportions specified. These pieces should be either cubes of not less than 4 inches each way, or cylinders not less than 6 inches diameter, and of a length not less than the diameter. They should be prepared in moulds, and punned as described for the work. Not less than four cubes or cylinders should be used for each test, which should be made twenty-eight days after moulding. The pieces should be tested by compression, the load being slowly and uniformly applied. The average of the results should be taken as the strength of the concrete for the purposes of calculation, and in the case of concrete made in proportions of 1 cement, 2 sand, 4 hard stone the strength should not be less than 1,800 lbs. per square inch. Such a concrete should develop a strength of 2,400 lbs. at 90 days.

Loading tests on the structure itself should not be made until at least two months have elapsed since the laying of the concrete. The test load should not exceed one and a half times the accidental load. Consideration must also be given to the action of the adjoining parts of the structure in cases of partial loading. In no case should any test load be allowed which would cause the stress in any part of the reinforcement to exceed two-thirds of that at which the steel reaches its elastic limit.

METHODS OF CALCULATION.

Data.

1. *Loads.*—In designing any structure there must be taken into account:—

(*a*) The weight of the structure.
(*b*) Any other permanent load, such as flooring, plaster, &c.
(*c*) The accidental or superimposed load.
(*d*) In some cases also an allowance for vibration and shock.

Of all probable distributions of the load, that is to be assumed in calculation which will cause the greatest straining action.

(i.) The weight of the concrete and steel structure may be taken at 150 lbs. per cubic foot.

(ii.) In structures subjected to very varying loads and more or less vibration and shock, as, for instance, the floors of public halls, factories, or workshops, the allowance for shock may be taken equal to half the accidental load. In structures subjected to considerable vibration and shock, such as floors carrying machinery, the roofs of vaults under passage ways and courtyards, the allowance for shock may be taken equal to the accidental load.

(iii.) In the case of columns or piers in buildings, which support three or more floors, the load at different levels may be estimated in this way. For the part of the roof or top floor supported, the full accidental load assumed for the floor and roof is to be taken. For the next floor below the top floor 10 per cent. less than the accidental load assumed for that floor. For the next floor 20 per cent. less, and so on to the floor at which the reduction amounts to 50 per cent. of the assumed load on the floor. For all lower floors the accidental load on the columns may be taken at 50 per cent. of the loads assumed in calculating those floors.*

Beams.

2. *Spans.*—These may be taken as follows :—For beams the distance from centre to centre of bearings ; for slabs supported at the ends, the clear span + the thickness of slab ; for slabs continuous over more than one span the distance from centre to centre of beams.

3. *Bending moments.*—The bending moments must be calculated on ordinary statical principles, and the beams or slabs designed and reinforced to resist these moments. In the case of beams or slabs continuous over several spans or fixed at the ends, it is in general sufficiently accurate to assume that the moment of inertia of the section has a constant value.

Where the maximum bending moments in beams or floor slabs continuous over three or more equal spans and under uniformly distributed loads, are not determined by exact calculation, the bending moments should not be taken less than $+\frac{wl^2}{12}$ at the centre of the span and $-\frac{wl^2}{12}$ at the intermediate supports.

When the spans are of unequal lengths, when the beam or slab is continuous over two spans only, or when the loads are not uniformly distributed, more exact calculations should be made.

If the bending moments are calculated by the ordinary theory of continuous beams it should be remembered that the supports are usually assumed level, and if this is not the case, or the supports sink out of level, the bending moments are altered.

4. *Stresses.*—The internal stresses are determined, as in the case of a homogeneous beam, on these approximate assumptions :—

---

* In the case of many warehouses and buildings containing heavy machines it is desirable not to make any reduction of the actual loads.

(*a*) The coefficient of elasticity in compression of stone or gravel concrete, not weaker than 1 : 2 : 4, is treated as constant and taken at one-fifteenth of the coefficient of elasticity of steel.

			lbs. per sq. in.
Coefficient for concrete	=	E_c =	2,000,000
„ „ steel	=	E_s =	30,000,000

$$\frac{E_s}{E_c} = 15.$$

It follows that at any given distance from the neutral axis, the stress per square inch on steel will be fifteen times as great as on concrete.

(*b*) The resistance of concrete to tension is neglected, and the steel reinforcement is assumed to carry all the tension.

(*c*) The stress on the steel reinforcement is taken as uniform on a cross-section, and that on the concrete as uniformly varying. In the case of steel of large section it may be necessary to consider the stress as varying across the section.

5. *Working stresses.*—If the concrete is of such a quality that its crushing strength is 1,800 lbs. per square inch after twenty-eight days as determined from the test cubes made in accordance with Clause 13, and if the steel has a tenacity of not less than 60,000 lbs. per square inch, the following stresses may be allowed :—

	lbs. per sq. in.
Concrete, in compression in beams subjected to bending	600
Concrete in columns under simple compression . .	600
Concrete in shear in beams	60
Adhesion * or grip of concrete to metal	100
Steel in tension	16,000
Steel in compression . . .	fifteen times the stress in the surrounding concrete
Steel in shear	12,000

When the proportions of the concrete differ from those stated above, the stress allowed in compression on the concrete may be taken at one-third the crushing stress of the cubes at twenty-eight days as determined above.

If stronger steel is used, the allowable tensile stress may be taken at one-half the stress at the yield point of the steel, but in no case should it exceed 20,000 lbs. per square inch.

Beams with Single Reinforcement.

Beams with single reinforcement can be divided into three classes :

(*a*) Beams of T form in which the neutral axis falls outside the slab.
(*b*) Beams of T form in which the neutral axis falls within the slab.
(*c*) Rectangular beams.

The equations found for (*a*) are general equations, from which the equations for (*b*) and (*c*) may be deduced.

In the calculation of all beams, the area upon which the ratio of tensile reinforcement is taken is considered as a rectangle of breadth equal to the greatest breadth of the beam and of depth equal to the greatest effective depth of the beam.

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* It is desirable that the reinforcing rods should be so designed that the adhesion is sufficient to resist the shear between the metal and concrete. Precautions should in every case be taken by splitting or bending the rod ends or otherwise to provide additional security against the sliding of the rods in the concrere.

In designing beams where the rib is monolithic with a slab, the beam may be considered to be of **T** form. The slab must first be calculated and designed having its own reinforcing bars transverse to the rib. The whole of the slab cannot in general be considered to form part of the upper flange of the **T** beams. The width b of the upper flange may be assumed to be not greater than one-third the span of the beams, or more than three-fourths of the distance from centre to centre of the reinforcing ribs or more than fifteen times the thickness of slab. The width b_r of the rib should not be less than one-sixth of the width b of the flange.

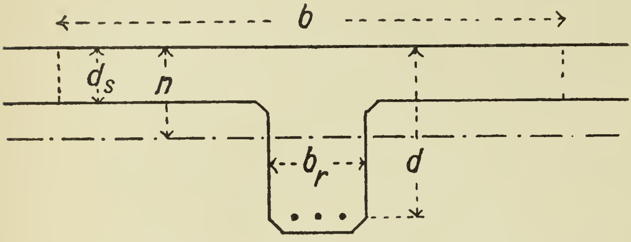

Fig. 1.

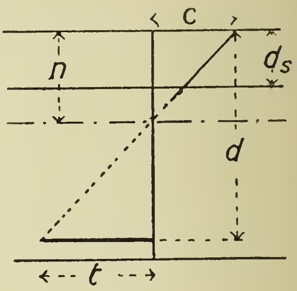

Fig. 2.

(*a*) Beams of **T** section where the neutral axis falls outside the slab.

In this case the small compression in the rib between the underside of the slab and the neutral axis may be neglected. In a homogeneous beam the stresses are proportional to the distances from the neutral axis. In a discrete beam, such as a beam of concrete and steel, on account of the greater rigidity of steel, at a given distance from the neutral axis the stress in the steel will be m times as great as in concrete.

Hence

$$\frac{mc}{t} = \frac{n_{,}d}{d(1-n_{,})} = \frac{n_{,}}{1-n_{,}}$$

$$\frac{c}{t} = \frac{n_{,}}{m(1-n_{,})}.$$

The mean compressive stress in the flange is

$$\tfrac{1}{2}\left\{c + c\,\frac{n-d_s}{n}\right\} = \frac{c}{2}\cdot\frac{2n-d_s}{n}$$

$$= c\left(1 - \frac{s_{,}}{2n_{,}}\right)$$

and the total compression is

$$bd_s\frac{c}{2}\cdot\frac{2n-d_s}{n}.$$

The area of reinforcement $A_t = rbd$ and the total tension is

$$trbd$$

Equating total compression and total tension

$$bd_s\frac{c}{2}\cdot\frac{2n-d_s}{n}=trbd$$

$$\frac{c}{t}=\frac{2rn_{,}}{(2n_{,}-s_{,})s_{,}}.$$

Equating these two values for $\frac{c}{t}$

$$\frac{n_{,}}{m(1-n_{,})}=\frac{2rn_{,}}{(2n_{,}-s_{,})s_{,}}$$

$$n_{,}=\frac{s_{,}^2+2mr}{2(s_{,}+mr)}.$$

The value of the lever arm is

$$d-\frac{d_s}{3}\,\frac{3n-2d_s}{2n-d_s}.$$

$$=d\left[1-\frac{s_{,}}{3}\left(\frac{3n_{,}-2s_{,}}{2n_{,}-s_{,}}\right)\right]$$

The compressive resistance moment of the beam is

$$R_c=cbdd_s\frac{(s_{,}^3+4mrs_{,}^2-12mrs_{,}+12mr)}{6(s_{,}^2+2mr)}.$$

The tensile resistance moment is

$$R_t=tbd^2\,\frac{(s_{,}^3+4mrs_{,}^2-12mrs_{,}+12mr)}{6m(2-s_{,})}.$$

To obtain stresses in the concrete and steel equal to c and t respectively r must have a value

$$\frac{2mcs_{,}-mcs_{,}^2-ts_{,}^2}{2mt}.$$

When r exceeds the value given by this equation the equation to R_c must be used in determining the moment of resistance. When r is less than the above value the equation to R_t must be used.

The following equation gives the value for r which causes the neutral axis to be at the underside of the slab

$$r=\frac{s_{,}^2}{2m(1-s_{,})}.$$

(*b*) When the neutral axis falls within the slab, or is at the bottom edge of the slab, the equations for values of n, R_c and R_t can be simplified and become

$$n_{,}=\sqrt{(m^2r^2+2mr)}-mr.$$

$$R_c=\frac{cbd^2n_{,}}{2}\left(1-\frac{n_{,}}{3}\right).$$

$$R_t=trbd^2\left(1-\frac{n_{,}}{3}\right).$$

To obtain stresses in the concrete and steel equal to c and t respectively

r must equal $\frac{mc^2}{2t(mc+t)}$.

(*c*) For rectangular beams, not of **T** form, the equations given for **T** beams under (*b*) apply.

The ratio of reinforcement may be taken on any other suitable sectional area if the formulæ are modified in accordance.

Slabs supported or fixed on more than two sides.

It does not appear that there is either a satisfactory theory or trustworthy experiments from which the strength of rectangular slabs supported or fixed on all four edges can be determined. [*See Appendices for a statement of some rules which have been used in determining the strength of slabs.*]

SHEAR REINFORCEMENT.

It is always desirable to provide reinforcement to resist the shearing and diagonal tension stresses in reinforced concrete beams. The diagonal tension stresses depend on the vertical and horizontal shear and also on the longitudinal tension at the point considered. As the longitudinal tension in the concrete at any given point is very uncertain, the amount and direction of the diagonal tension cannot be exactly determined.

It is the general practice to determine the necessary reinforcement by taking the vertical and horizontal shearing only into consideration.

The following equations may be used to determine the necessary resistance to shearing.

When S, the total shear in lbs. at a vertical section, does not exceed $60ba$, no shear reinforcement is required.*

When S exceeds $60ba$, vertical shear members may be provided to take the excess and proportioned by the following rule : –

$$\frac{A_s \, . \, s_s \, . \, a}{p} = S - 60ba$$

or

$$A_s = \frac{(S - 60ba)\,p}{a\, s_s};$$

where s_s is the unit resistance of the steel to shearing, and p is the pitch, or distance apart of the vertical shear members or groups of shear members, of area A_s.

In the case of **T** beams, b_r should be substituted for b.

In important cases, when extra security is required, the resistance of the concrete to shear, represented by $60ba$, should be disregarded.

When the shear members are inclined at an angle of about 45° to the horizontal, the area A_s may be decreased in the proportion of $\frac{1}{\sqrt{2}}$.

These equations, though based on somewhat uncertain assumptions, give reasonable results. But experience shows that :

(*a*) In general, floor slabs require no special reinforcement against shearing, and that the bending up of alternate bars near the end is sufficient.

---

* The value of Sp is shown in the appendix to be $\frac{B_1 - B_2}{a}$

(*b*) In beams, especially in **T** beams, shearing reinforcement should be provided at distances apart not exceeding the depth of the beam.

(*c*) It is desirable to bend up one or more of the bars of the tension reinforcement near the supports. When bent at an angle of about 45° the effect of this may be taken into account in the manner set out above; when bent at a small angle to the horizontal the effect is very indeterminate.

(*d*) As the resistance of the shear members to the pull depends on the adhesion, and the anchorage at the ends, it is desirable to use bars of small diameter, and to anchor the stirrups at both their ends. In all cases the stirrups must be taken well beyond the centre of compression.

PILLARS AND PIECES UNDER DIRECT THRUST.

Definitions.

The length is to be measured between lateral supports (neglecting ordinary bracketing).

The effective diameter of a pillar means the least width and should be measured to the outside of the outermost vertical reinforcement.

The effective area of a pillar means the area contained by the outermost lateral reinforcement, and should be measured to the outside of the outermost vertical reinforcement.

Loading and Length of Pillars.

If the load is strictly axial the stress is uniform on all cross-sections.

Lateral bending of the pillar as a whole is not to be feared provided:

(*a*) That the ratio of length to least outside diameter does not exceed 18.

(*b*) That the stress on the concrete does not exceed the permissible working stress for the given pillar.

(*c*) That the load be central.

(*d*) That the pillar be laterally supported at the top and base.

Construction.

Lateral reinforcement properly disposed raises the ultimate strength and increases the security against sudden failure, by preventing the lateral expansion of the concrete and the sudden disruption of the pillar.

Practical considerations lead to the addition of longitudinal bars, and the formation of an enveloping network of steel.

The total cross-sectional area of the vertical reinforcement should never be less than 0·8 per cent. of the area of the hooped core.

There should be at least six vertical bars when curvilinear laterals are used, and four for square pillars having rectilinear laterals.

In the case of rectangular pillars in which the ratio between the greater and the lesser width (measured to the outside of the vertical bars) exceeds one and a half, the cross-section of the pillars should be subdivided by cross ties; and the number of vertical bars should be such that the distance between the vertical bars along the longer side of the rectangle should not exceed the distance between the bars along the shorter side of the rectangle.

The most efficient disposition of the lateral reinforcement would appear to be in the form of a cylindrical helix, the pitch or distance between the coils being small enough to resist the lateral expansion of the concrete.

Jointed circular hoops as ordinarily made are apparently not quite so efficient.

Rectilinear ties are still less adapted to resist the lateral or radial expansion of a highly stressed core.

The volume of curvilinear laterals should never be less than 0·5 per cent. of the volume of hooped core.

The diameter of rectilinear laterals should not be less than $\frac{3}{16}$ of an inch.

STRENGTH.

The amount of the increase of strength in hooped pillars depends upon

1. The *form* of hooping (whether curvilinear or rectilinear, &c.).
2. The *spacing* or distance between the hoops.
3. The *quantity* of hooping relative to the quantity of concrete in the core of the pillar.
4. The *quality of the concrete.*

Consequently the increase of strength may be shown to be equal to the product of the four factors ($u.f.s.r$).

u = the *ultimate* compressive stress on concrete not hooped (per unit of area).
f = a *form* factor or constant which will vary according to whether the hooping is curvilinear or rectilinear, &c.
s = *Spacing* factor or constant which will vary with the pitch of the laterals.
V_h = *Volume* of hooped reinforcement in cubic inches.
V = *Volume* of hooped core in cubic inches.
$r = V_h/V$ = the *ratio* of volumes, i.e. the *ratio* of the volume of *helical* or horizontal reinforcement to the volume of hooped core.

The *ultimate* compressive stress on concrete *not hooped* being $= u$, and the increase of strength due to hooping being

$$u.f.s.r,$$

the total resistance of the hooped material per unit of area will then be

$$= u + u.f.s.r$$
$$= u\,[1 + f.s.r].$$

Let $c_p =$ the working *compressive* stress on a *prism* of concrete (not hooped) $= W_F u$

$$W_F = \text{the working factor} = c_p/u,$$

Then the safe compressive stress on the hooped core $= c$, where

$$c = W_F u[1 + f.s.r]$$
$$= c_p[1 + f.s.r]$$

The values of f, s and their product may be obtained from the following table :—

Form of Lateral Reinforcement	Form Factor $= f$	Spacing of Laterals in Terms of Diameter of Hooped Core	Spacing Factor $= s$	Value of $f\,s$
Helical	1	$0{\cdot}2d$	32	32
,, ...	1	$0{\cdot}3d$	24	24
,, ...	1	$0{\cdot}4d$	16	16
Circular Hoops .	0·75	$0{\cdot}2d$	32	24
,, ,, .	0·75	$0{\cdot}3d$	24	18
,, ,, .	0·75	$0{\cdot}4d$	16	12
Rectilinear . . .	0·5	$0{\cdot}2d$	32	16
,, . .	0·5	$0{\cdot}3d$	24	12
,, . .	0·5	$0{\cdot}4d$	16	8
,, . .	0·5	$0{\cdot}5d$	8	4
,, . .	0·5	$0{\cdot}6d$	0	0

Let p = the pitch of the laterals in inches [*i.e.* the axial spacing of the laterals].

d = the effective diameter of the hooped core in inches.

The spacing factor should not be taken at more than 32, even if p is less than $\cdot 2d$, but intermediate values of the spacing factor may be obtained from the equation

$$s = 48 - 80\frac{p}{d}$$

It will be seen from the above table that the advantage of hooping disappears with an increase in the spacing of the laterals, irrespective of the volume of hooping or the value of r.

Before the safe stress on the hooped core can be obtained it will be necessary to give values to W_F and u. A table for this purpose will be found below.

The value of the working compressive stress on the concrete of the hooped core having been obtained, the maximum permissible pressure or load may be obtained from the equation

$$P = c\,[A + (m - 1)\,A_V], \text{ where}$$

A = the effective *area* of the pillar.

$m = \dfrac{E_s}{E_c}$ = *modular* ratio.

A_V = *Area* of *vertical* reinforcement.

P = total safe *pressure* on pillar.

WORKING STRESSES.

A safety factor of 4 at 90 days is recommended for all pillars.

The following table of working stresses is suitable if good materials are used, and is based on the assumption that test cubes have *at least* the strength given at the periods stated :—

Table showing the Value of u and c_p for Pillars.

Proportions of Concrete Measured by Volume	Pounds of Cement to $13\frac{1}{2}$ cubic feet of Sand and 27 cubic feet of Shingle or Broken Stones	Value of u at 28 days in pounds per square inch	Value of u at 90 days in pounds per square inch	Value of c_p at 90 days in pounds per square inch (Safety factor = 4) (Working factor = 1/4)
1 : 2 : 4	610	1,800	2,400	600
1 : $1\frac{1}{2}$: 3	810	2,100	2,800	700
1 : 1 : 2	1,220	2,700	3,600	900

It is assumed that the tests of the strength of the concrete are made on unrammed cubes and of the same consistency as the concrete used on the work.*

LIMITATIONS OF STRESS ON PILLARS.

The following limits of stress should be observed in pillars :

(*a*) The stress on the metal reinforcement (*i.e.*, the value of $m.c$) should not exceed 0·5 of the yield point of the metal ;

(*b*) Whatever the percentage of lateral reinforcement the working stress on the concrete of pillars should not exceed $(0{\cdot}34 + 0{\cdot}32\,f)\,u$ where

f = *form* factor.

u = *ultimate crushing* resistance of the concrete.

Form of Laterals	Form Factor	Value of $(0{\cdot}34 + 0{\cdot}32f)\,u$
Rectilinear	0·5	$0{\cdot}5\,u$
Independent Circular Hoops . .	0·75	$0{\cdot}58\,u$
Helical	1·00	$0{\cdot}66\,u$

If these limits are adopted, the working stress on hooped concrete will always fall within the "limit of continued endurance" for plain concrete.

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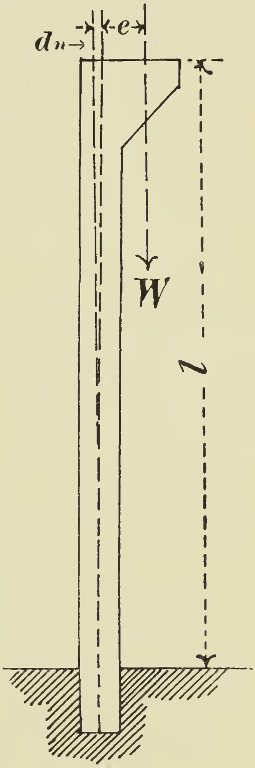
* The limit of 2,400 pounds per square inch given in the previous report of the Committee was adopted on the assumption that the cubes would be rammed with iron rammers under laboratory conditions.

PILLARS ECCENTRICALLY LOADED.

If a pillar initially straight is loaded eccentrically, as when a beam rests on a bracket attached to the pillar, it may be regarded as fixed at the base and free at the loaded end. Then it must bend in the plane passing through the load, the *deflection* at the top being d_n. Let e be the *eccentricity* of the load measured from the centre of the pillar when straight. Then the bending moment at the base of the pillar is $W(d_n + e)$. But it is known that d_n will be small compared with e, provided that W is small compared with $2EI/l^2$, and this will be the case in such conditions as are likely to occur in designing concrete pillars. Then the bending moment may be taken as We, and the extreme "*fibre*" stress at the edge of the base of the pillar, treating it as homogeneous, will be

$$f = W\left\{\frac{1}{A} \pm \frac{e}{S_m}\right\}$$

very nearly, where A is the whole section of the pillar and S_m the *section modulus* relatively to an axis through the centre of gravity and at right angles to the plane of bending.



In dealing with reinforced pillars which are not homogeneous, it is convenient to substitute for the actual section of the pillar what may be termed the equivalent section, or section of concrete equivalent in resistance to the actual pillar. If A is the effective area of section of the pillar (including the area of reinforcement), and A_V is the *area* of *vertical* reinforcement, then the equivalent section is

$$A_E = A + (m - 1)A_V.$$

If d is the *depth* of the section in the plane of bending, the Inertia moment relatively to the neutral axis can be expressed in the form

$$I = nAd^2,$$

and the section modulus in the form $S_m = 2nAd$. (*See Appendix V.*)

It is desirable in pillars that there should be no tension, and generally when the vertical load is considerable there is none. Cases in which the eccentricity is so great that there is tension must be treated by the methods applicable to beams if it is made a condition that the steel carries all the tension. In the following cases it is assumed that there is no tension.

Case I.—Pillar of Circular Section, Reinforcements Symmetrical and Equidistant from the Neutral Axis. Let m be the modular ratio $= E_s/E$, A the effective cross-section of the column in square inches, A_V the area of vertical reinforcement in square inches, d the *diameter* of the pillar, d_v the *distance* between the *vertical* reinforcing bars perpendicular to the neutral axis. Then the equivalent section is

$$A_E = A + (m - 1)A_V,$$

and the section modulus is (*Appendix V.*)

$$S_m = \tfrac{1}{8}Ad + \tfrac{1}{2}(m - 1)A_V\frac{d_v^2}{d}.$$

The stress at the edges of the section can then be calculated by the general equation

$$f = W\left\{\frac{1}{A} \pm \frac{e}{S_m}\right\}$$

where e is the eccentricity of the load in inches and W the weight or load in pounds. The greater value of stress must not exceed the safe stress stated above.

Case II.—Rectangular Section with Reinforcement Symmetrical and Equidistant from the Neutral Axis. Using the same notation as in the last case, d being now the *depth* of the section in the plane of bending, the section modulus is (*Appendix V.*)

$$S_m = \tfrac{1}{6}Ad + \tfrac{1}{2}(m - 1)A_V\frac{d_v^2}{d},$$

and the stresses are given by the same equation as in the previous case.

Case III.—Column of Circular Section with Reinforcing Bars arranged in a Circle.—Using the same notation as in Case I., h_t being the diameter of the circle of reinforcing bars, the section modulus is (*Appendix V.*)

$$S_m = \tfrac{1}{8}Ad + \tfrac{1}{4}(m - 1)A_V\frac{d_v^2}{d},$$

and the stresses are given by the same equation as in Case I.

(c) *Long Pillars axially loaded.*

For pillars more than 18 diameters in length there is risk of lateral buckling of the pillar as a whole. The strength of such pillars would be best calculated by Gordon's formula, but there are no experiments on long pillars by which to test the values of the constants for a concrete or concrete and steel pillar. There does not seem, however, to be any probability of serious error if the total load is reduced in a proportion inferred from Gordon's formula to allow for the risk of buckling.

Let, as before, A = the *area* of the column in inches; A_V = the *area* of *vertical* reinforcement. Then $A_E = A + (m - 1)A_V$ is the equivalent section. Let N be the *numerical* constant in the equation, $I = NAd^2$ (*Appendix V.*), and d the least diameter of the pillar.

Then for a pillar fixed in direction at both ends Gordon's formula is

$$\frac{W}{Au} = \frac{1}{1 + \dfrac{l^2}{C_1Nd^2}} = \frac{1}{1 + C_2'}$$

so that the pillar will carry less than a short column of the same dimensions in the ratio of $1 + C_2$ to 1, or, in other words, the column will be safe if calculated as a short column, not for the actual weight or pressure P, but for a weight or pressure $= (1 + C_2)W$.

The constant C_1 has not been determined experimentally for reinforced long columns. But its probable value is

$$C_1 = \frac{4\pi^2 E_C}{u}$$

where u is the *ultimate* crushing stress. Putting $E_C = 2{,}000{,}000$ and $u = 2{,}500$, then $C_1 = 32{,}000$. Looking at the well-understood uncertainty of the rules for long columns, very exact calculation is useless. Some values of N for ordinary types of column are given in Appendix V. Taking these values, the following are the values of $1 + C_2$:—

VALUES OF $1 + C_2$.

	Case I.	*Case II.*	*Case III.*
$\frac{l}{d}$	N = 0·098	N = 0·075	N = 0·0646
20	1·13	1·17	1·19
25	1·20	1·26	1·30
30	1·29	1·38	1·44

The differences of $1 + C_2$ for considerable differences of N are not very great. In any case N can be found by the method in the Appendix with little trouble.

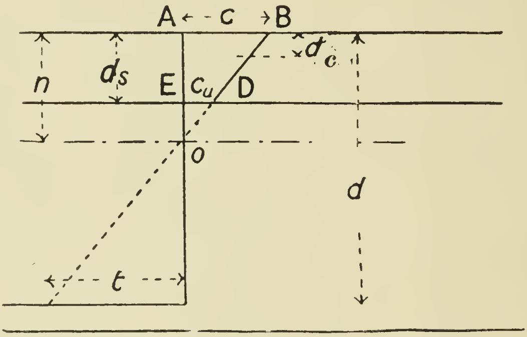
In the case of columns fixed at one end and rounded or unfixed at the other, $2C_2$ must be substituted for C_2. If the column is rounded at both ends, $4C_2$ must be substituted for C.

APPENDIX I.

CALCULATIONS FOR SINGLY REINFORCED BEAMS

By Capt. J. G. Fleming, R.E.

To find the moment of resistance of a **T** beam in which the neutral axis falls outside the slab



The total compression is equal to the area ABDE × width of flange.

$$\text{The area ABDE} = \frac{\text{AB}+\text{DE}}{2} \times \text{AE}.$$

$$= \frac{c+c_u}{2} d_s.$$

$$\text{AB} = c$$

$$c_u = \text{DE} = \frac{\text{AB} \,.\, \text{OE}}{\text{OA}}.$$

$$= \frac{c \,.\, (n-d_s)}{n}.$$

$$\text{AE} = d_s.$$

$$\text{area ABDE} = \left(c + \frac{c(n-d_s)}{n}\right)\frac{d_s}{2}.$$

$$= c \,.\, \frac{(2n-d_s)}{n} \,.\, \frac{d_s}{2}.$$

$$\text{Then the total compression } = \frac{c(2n-d_s)}{n} \,.\, \frac{d_s}{2} \,.\, b.$$

The area of tensile reinforcement $A_t = rbd$.
The total tension $= T = trbd$.

Equating total compression to total tension

$$c\,.\,\frac{(2n-d_s)}{n}\,.\,\frac{d_s}{2}\,.\,b=trbd.$$

Let $c_{,}=$ ratio of c to t, then

$$c_{,}=\frac{rbd\,2n}{(2n-d_s)d_sb}.$$

$$=\frac{rd\,2n}{(2n-d_s)d_s}.$$

Substituting $n_{,}d$ for n and $s_{,}d$ for d_s.

$$c_{,}=\frac{r\,d\,2n_{,}d}{(2n_{,}d-s_{,}d)\,.\,s_{,}d}.$$

$$=\frac{2rn_{,}d^2.}{(2n_{,}-s_{,})\,s_{,}d^2}.$$

$$=\frac{2rn_{,}}{(2n_{,}-s_{,})s_{,}}.$$

It has already been shown on page 12 of the report that another value for c/t can be obtained, deduced from the supposition that the stress on any layer is jointly proportional to its distance from the neutral axis and to the rigidity of the material.

This value for c/t is $\frac{n_{,}}{m(1-n_{,})}$.

Equating these two values for c/t,

$$\frac{n_{,}}{m(1-n_{,})}=\frac{2rn_{,}}{(2n_{,}-s_{,})s_{,}},$$

whence

$$n_{,}=\frac{s_{,}^2+2mr}{2(s_{,}+mr)}.$$

It has been found that $c/t=\frac{2rn_{,}}{(2n_{,}-s_{,})s}$.

To express this in terms of m, r and s,

$$n_{,}=\frac{s_{,}^2+2mr}{2(s_{,}+mr)}$$

substituting for $n_{,}$ in the equation for c/t

$$c/t=\frac{2r\,\frac{s_{,}^2+2mr}{2(s_{,}+mr)}}{\left\{\frac{2(s_{,}^2+2mr)}{2(s_{,}+mr)}-s_{,}\right\}s_{,}}$$

$$=\frac{2r(s_{,}^2+2mr)}{(2s_{,}^2+4mr-2s_{,}^2-2mrs_{,})s_{,}}$$

$$=\frac{2r(s_{,}^2+2mr)}{2mr(2-s_{,})s_{,}}$$

$$=\frac{s_{,}^2+2mr}{m(2-s_{,})s_{,}}$$

To find the moment of resistance it is first necessary to find the lever arm.

The centre of pressure is at a distance

$$\frac{d_s}{3}\left(\frac{2c_{,,}+c}{c_{,,}+c}\right)$$

from the extreme fibre in compression.

$$c_{,,}=\frac{c(n-d_s)}{n}.$$

$$d_c=\frac{d_s}{3}\left(\frac{\frac{2c(n-d_s)}{n}+c}{\frac{c(n-d_s)}{n}+c}\right)$$

$$=\frac{d_s}{3}\,.\left[\frac{3cn-2cd_s}{2cn-cd_s}\right]$$

$$=\frac{d_s}{3}\,.\left(\frac{3n-2d_s}{2n-d_s}\right).$$

The lever arm is

$$a=d-d_c$$

$$=d-\frac{d_s}{3}\,.\left(\frac{3n-2d_s}{2n-d_s}\right).$$

$$=\frac{6nd-3dd_s-3nd_s+2d_s^2}{3(2n-d_s)}.$$

expressing n and d_s as $n_{,}d$ and $s_{,}d$

$$a=\frac{6n_{,}d^2-3s_{,}d^2-3n_{,}s_{,}d^2+2s_{,}^2d^2}{3d(2n_{,}-s_{,}).}$$

$$=\frac{d(6n_{,}-3s_{,}(1+n_{,})+2s_{,}^2)}{3(2n_{,}-s_{,})}.$$

The total compression

$$=\frac{c(2n-d_s)}{n}\,.\,\frac{d_s}{2}\,.\,b$$

$$=\frac{cbd_s}{2}\,.\left[\frac{2n-d_s}{n}\right]$$

$$=\frac{cbd_s}{2}\,.\,\frac{d(2n_{,}-s_{,})}{d\,.\,n_{,}}.$$

$$=\frac{cbd_s}{2}\,.\,\frac{(2n_{,}-s_{,})}{n_{,}}.$$

Then the *resistance* moment in terms of the *compressive* stress is found by the equation

$$R_c=\frac{cbd_s}{2}\frac{(2n_{,}-s_{,})}{n}a.$$

$$=\frac{cbd_s}{2}\,.\,\frac{(2n_{,}-s_{,})}{n_{,}}\,.\,d\,\frac{(6n_{,}-3s_{,}(1+n_{,})+2s_{,}^2)}{3(2n_{,}-s_{,})}.$$

$$=\frac{cbdd_s\,.\,(6n_{,}-3s_{,}(1+n_{,})+2s_{,}^2)}{6n_{,}}$$

Expressing $n_{,}$ in terms of $s_{,}$, m and r

$$\mathrm{R}_c = \frac{cbdd_s}{6}\left\{\frac{6\cdot\frac{(s_{,}^2+2mr)}{2(s_{,}+mr)} - 3s_{,}\left(1+\frac{s_{,}^2+2mr}{2(s_{,}+mr)}\right)+2s_{,}^2}{\frac{s_{,}^2+2mr}{2(s_{,}+mr)}}\right)$$

$$= \frac{cbdd_s}{6}\cdot\left[\frac{6s_{,}^2+12mr-6s_{,}^2-12mrs_{,}-3s_{,}^3+4s_{,}^3+4mrs_{,}^2}{s_{,}^2+2mr}\right]$$

$$= \frac{cbdd_s}{6}\left[\frac{s_{,}^3+4mrs_{,}^2-12mrs_{,}+12mr}{s_{,}^2+2mr}\right].$$

The *resistance* moment in terms of the *tensile* stress is found by the equation

$$\mathrm{R}_t = trbda$$

$$= \frac{trbd^2(6n_{,}-3s_{,}(1+n_{,})+2s_{,}^2)}{3(2n_{,}-s_{,})}.$$

Expressing $n_{,}$ in terms of s, m and r

$$\mathrm{R}_t = trbd^2\cdot\left(\frac{6\cdot\frac{s_{,}^2+2mr}{2(s_{,}+mr)} - 3s_{,}\left(1+\frac{s_{,}^2+2mr}{2(s_{,}+mr)}\right)+2s_{,}^2}{\left\{\frac{2\cdot(s_{,}^2+2mr)}{2(s_{,}+mr)}-s_{,}\right\}}\right)$$

$$= tbd^2\left(\frac{s_{,}^3+4mrs_{,}^2-12mrs_{,}+12mr}{6m(2-s_{,})}\right).$$

To find the ratio of reinforcement to give stresses of c and t in the concrete and steel.

$$\mathrm{R}_c = \mathrm{R}_t.$$

$$\frac{cbdd_s}{6}\;\frac{(s_{,}^3+4mrs_{,}^2-12mrs_{,}+12mr)}{(s_{,}^2+2mr)}$$

$$= \frac{tbd^2(s_{,}^3+4mrs_{,}^2-12mrs_{,}+12mr)}{6m(2-s_{,})}.$$

$$\frac{cd_s}{s_{,}^2+2mr} = \frac{td}{m(2-s_{,})}.$$

$$cs_{,}m(2-s_{,}) = t(s_{,}^2+2mr).$$

$$2mcs_{,}-mcs_{,}^2 = ts_{,}^2+2mtr.$$

$$r = \frac{2mcs_{,}-mcs_{,}^2-ts_{,}^2}{2mt}.$$

To find the value of r such that the neutral axis is at the underside of the slab.

$$d_s = n$$

$$s_{,} = n_{,} = \frac{s_{,}^2+2mr}{2(s_{,}+mr)}.$$

$$2s_{,}^2+2mrs_{,} = s_{,}^2+2mr.$$

$$2mr(1-s_{,}) = s_{,}^2$$

$$r = \frac{s_{,}^2}{2m(1-s_{,})}.$$

When the neutral axis falls at the underside of the slab or within the slab $n_{,}=s_{,}$.

The equation

$$n_{,}=\frac{s_{,}^{2}+2mr}{2(s_{,}+mr)}$$

becomes

$$n_{,}=\frac{n_{,}^{2}+2mr}{2(n_{,}+mr)}$$

$$2n_{,}^{2}+2mrn_{,}=n_{,}^{2}+2mr$$

$$n_{,}=\sqrt{m^{2}r^{2}+2mr}-mr.$$

The equation

$$\mathrm{R}_{c}=\frac{cbdd_{s}(6n_{,}-3s_{,}(1+n_{,})+2s_{,}^{2})}{6n_{,}}$$

becomes

$$\mathrm{R}_{c}=cbdn_{,}d\frac{(6n_{,}-3n_{,}-3n_{,}^{2}+2n_{,}^{2})}{6n_{,}}$$

$$=\frac{cbd^{2}}{2}n_{,}\left(1-\frac{n_{,}}{3}\right).$$

The equation

$$\mathrm{R}_{t}=\frac{trbd^{2}(6n_{,}-3s_{,}(1+n_{,})+2s_{,}^{2})}{3(2n_{,}-s_{,})}$$

becomes

$$\mathrm{R}_{t}=\frac{trbd^{2}(6n_{,}-3n_{,}(1+n_{,})+2n_{,}^{2})}{3n_{,}}$$

$$=trbd^{2}\left(1-\frac{n_{,}}{3}\right).$$

To find the value of r to give values of c and t—

$$\frac{c}{t}=\frac{s_{,}^{2}+2mr}{m(2-s_{,})s_{,}}$$

$$=\frac{2r(s_{,}^{2}+2mr)}{(4mr-2mrs_{,})s_{,}}$$

$$=\frac{2r(s_{,}^{2}+2mr)}{(4mr+2s_{,}^{2}-2s_{,}^{2}-2mrs_{,})s_{,}}$$

$$=\frac{2r(s_{,}^{2}+2mr)}{[2(s_{,}^{2}+2mr)-2s_{,}(s_{,}+mr)]s_{,}}$$

$$=\frac{2r\frac{(s_{,}^{2}+2mr)}{2(s_{,}+mr)}}{\left[2\frac{(s_{,}^{2}+2mr)}{2(s_{,}+mr)}-2s_{,}\frac{(s_{,}+mr)}{2(s_{,}+mr)}\right]s_{,}}$$

$$=\frac{2rn_{,}}{(2n_{,}-s_{,})s_{,}}$$

and if $s_{,} = n_{,}$

$$\frac{c}{t} = \frac{2r}{n_{,}}$$

$$\therefore \quad r = \frac{cn_{,}}{2t}$$

$$\text{Also } \frac{c}{t} = \frac{n_{,}}{m(1 - n_{,})}$$

$$mc - mcn_{,} = n_{,}t$$

$$n_{,}(mc + t) = mc$$

$$n_{,} = \frac{mc}{mc + t}.$$

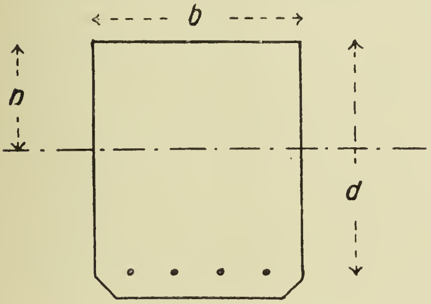
Substituting in the equation

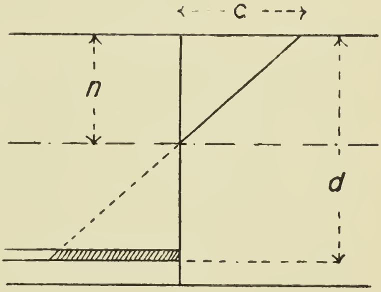
$$r = \frac{cn_{,}}{2t}$$

$$r = \frac{mc^2}{2t(mc + t)}.$$

The equations found for a T beam where the neutral axis does not fall outside the slab apply equally to rectangular beams.

These equations for rectangular beams may be equally easily deduced from first principles as outlined below.





The equation

$$\frac{c}{t} = \frac{n_{,}}{m(1 - n_{,})}$$

applies equally to this type of beam as to a T beam, depending entirely on the stress in any fibre varying directly with its distance from the neutral axis and the rigidity of the material.

The total compression $= \frac{1}{2}cbdn_{,}$.

The total tension $= trbd$.

Equating total compression and total tension

$$\frac{c}{t} = \frac{2r}{n_{,}}.$$

Equating these two values for $\frac{c}{t}$

$$\frac{n_{,}}{m(1 - n_{,})} = \frac{2r}{n_{,}}.$$

$$n_,^2 = 2mr - 2mrn_,$$

$$n_,^2 + 2mrn_, + m^2r^2 = 2mr + m^2r^2.$$

$$n_, + mr = \sqrt{m^2r^2 + 2mr}.$$

$$n_, = \sqrt{m^2r^2 + 2mr} - mr.$$

The resistance moment with regard to the concrete under compression is found by the equation

$$R_c = \text{total compression} \times \text{lever arm}$$

$$\text{The lever arm } a = d - \frac{n}{3}.$$

$$= d - \frac{n_,d}{3}.$$

$$= d\left(1 - \frac{n_,}{3}\right).$$

$$\therefore \qquad R_c = \frac{cbd}{2} n_, \,.\, d\left(1 - \frac{n_,}{3}\right)$$

$$= \frac{cbd^2}{2} n_, \left(1 - \frac{n_,}{3}\right).$$

The resistance moment with regard to the tensile stress on the steel

$$R_t = \text{total tension} \times \text{lever arm}$$

$$= trbd^2\left(1 - \frac{n_,}{3}\right).$$

To find the value for r to give values of c and t—

$$\frac{c}{t} = \frac{n_,}{m(1 - n_,)}.$$

$$mc - mcn_, = tn_,$$

$$n_,(mc + t) = mc$$

$$n_, = \frac{mc}{mc + t}$$

Also by equating total compression and total tension

$$\frac{c}{t} = \frac{2r}{n_,}$$

$$2rt = cn_,$$

$$r = \frac{cn_,}{2t}$$

substituting for $n_,$ its value $\dfrac{mc}{mc + t}$

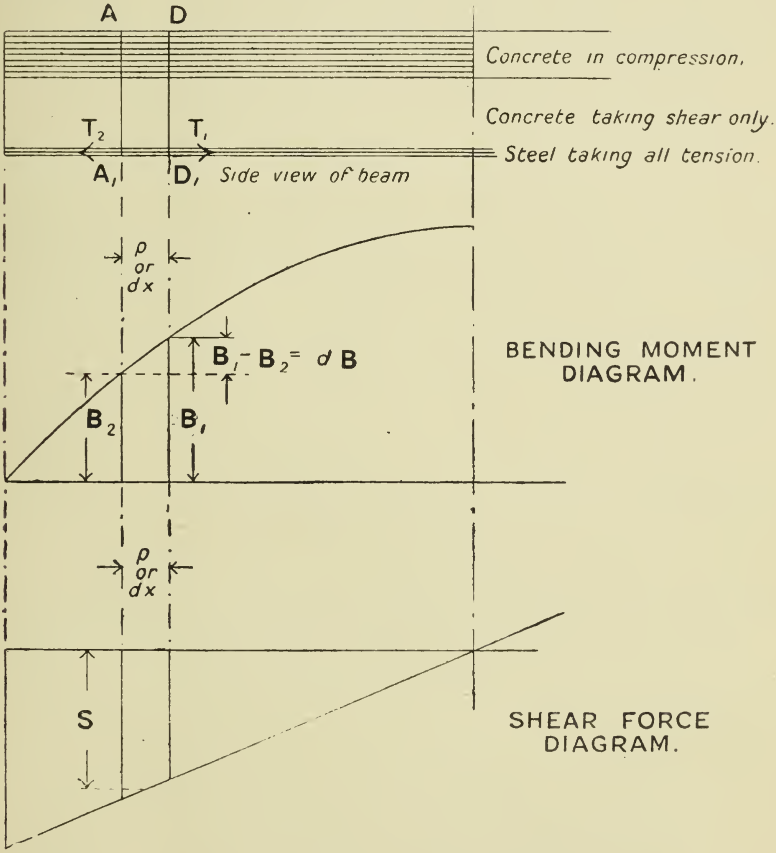
$$r = \frac{mc^2}{2t(mc + t)}$$

APPENDIX II.

SHEAR STRESSES IN REINFORCED CONCRETE BEAMS.

By W. Dunn.

In a reinforced concrete beam the steel reinforcement is assumed to take all the tension.



Accordingly, if B_2 is the bending moment at a section AA_1, and B_1 is the bending moment at a section DD_1, distant p from AA_1, the total stress on the steel at the former section is $\frac{B_2}{a}$ and at the latter section is $\frac{B_1}{a}$ where a is the lever arm.

The difference $T_1 - T_2 = \frac{B_1 - B_2}{a}$ is communicated from the steel to the concrete by adhesion, and produces a shearing stress in the concrete above it, which is distributed over an area of pb. If s is the unit shearing resistance of the concrete, we have

$$T_1 - T_2 = \frac{B_1 - B_2}{a} = pbs.$$

If the concrete cannot safely resist this unit shearing stress s, we must add steel shear members. If we add vertical steel shear members, let the sectional area of steel in the length p equal A_s and the safe unit shearing resistance of the steel be s_s. This area A_s may be provided in one member or in a group of members.

Then we must have

$$pbs + A_s s^s = \frac{B_1 - B_2}{a}.$$

The ratio in which the shear stress is distributed between the steel and concrete has not yet been determined, for which reason and where greater safety is desired, the shearing resistance of the concrete is often neglected and the vertical shear members determined by the formula

$$A_s s_s = \frac{B_1 - B_2}{a}.$$

Now $B_1 - B_2$ is the difference between the bending moments in the length p. From the principles of statics, this difference equals the average shearing force on the length p multiplied by the length p; that is, from the ordinary equation $\frac{dB}{dx} = S$, we get (since $dB = B_1 - B_2$ and $dx = p$)

$$B_1 - B_2 = Sp.$$

We may then put

$$pbs + A_s s_s = \frac{B_1 - B_2}{a} = \frac{Sp}{a}.$$

If the resistance of the concrete is neglected, we have

$$A_s s_s = \frac{Sp}{a}.$$

These rules fix the sectional area and spacing of the vertical shear members.

If there is a shear stress (and a shear stress only) on one face of a body, there must (by Rankine's theory of internal stress) be a shear stress of the same intensity (and a shear stress only) on any plane at right angles to the first plane.

Also, on the plane inclined at 45° in one direction to the first plane there must be a normal tension only, of the same intensity, and on the plane inclined at 45° in the other direction a normal compression only, of the same intensity. When p is the horizontal distance between shear members inclined at 45°, the extent of the planes, on which these normal tensions and compressions act, is less than the planes on which the shears act, in the ratio of 1 to $\sqrt{2}$ and the *amount* of the normal tension or compression to be taken by the shear member is also less in the ratio $\frac{1}{\sqrt{2}}$.

Accordingly, when shear members are inclined at 45° to the horizontal, and the tensile resistance of the concrete is neglected, the diagonal shear members are proportioned by the rule

$$A_s s_s = \frac{Sp}{a\sqrt{2}}$$

when the resistance of the concrete is neglected, and

$$rbs + A_s s_s = \frac{Sp}{a\sqrt{2}}$$

when the resistance of the concrete is taken into account.

APPENDIX III.

THE STRENGTH OF RECTANGULAR SLABS.

By E. F. ETCHELLS.

Grashof's formula for the maximum bending moment on flat plates is perfectly general in form. It is based *inter alia* upon the identity of the deflection (at the point of intersection) of any two cross-sections at right angles to each other.

Applying this principle to a rectangular slab, we see that a point at the centre of the area must have the same deflection along the major as along the minor axis.

The curvature will, however, be greater in the direction of the minor axis.

As the bending moment and the stress are both proportional to the curvature, it follows that the bending moment and the stress will both be greater in the direction of the minor axis.

Rankine's formula for the strength of rectangular plates is in form identical with Grashof's.

The Progress Report of the American Joint Committee on Concrete, presented to the American Society of Civil Engineers on January 20, 1909, gives the Grashof-Rankine rule for slabs.

The Austrian Government Report, 1908, adopts the Grashof-Rankine rule.

The French Government adopt an empirical formula by which the bending moment at the centre of a square slab fixed in direction on four sides and uniformly loaded =

$$\frac{Wl}{72} \text{ as against } \frac{W1}{48} \text{ obtainable}$$

by the Grashof-Rankine rule.

The references in the French Government Reports are as follows :—

" S'il s'agit d'un hourdis porté par deux cours de nervures orthogonales, d'écartements respectifs $l\ l'$, pour calculer le moment de flexion dans le sens de la portée l, on pourra, *faute de mieux,* le calculer comme si les nervures de portée l existaient seules, en multipliant le chiffre obtenu par le coefficient de reduction :

$$\frac{1}{1+2\frac{l^4}{l'^4}}$$

" On fera de même en permutant les lettres l et l' pour obtenir le moment de flexion dans le sens de la portée l'."

Another form of this equation for the bending moment reduction factor is also given in the French report :—

“ Hourdis portés par deux cours de poutres orthogonaux. On représente par l et l' les espacements, d’axe en axe, des poutres des deux cours orthogonaux.

“ Pour calculer le moment de flexion dans le sens de la portée l, on determine la valeur qu’il aurait si le second cours de poutres n’existait pas et on la multiplie par le coefficient de réduction :

$$\frac{l'^4}{l'^4 + 2l^4}$$

“ On procède de même pour calculer le moment de flexion dans le sens de la portée l' en remplaçant dans la formule, l par l' et l' par l.”

No other reason or proof of the French rule is given, and it would appear to be based on a limited range of tests and gives practically the average bending moment for the whole of the cross-section of a slab.

It is considered that it would be inadvisable to form conclusions as to the behaviour of slabs outside the limited range of a few experiments.

Moreover, it is considered injudicious to use formulæ for the average bending moment in lieu of the maximum bending moment on slabs which may measure as much as 30 feet by 60 feet and have a thickness of about 9 inches.

The Grashof-Rankine rule gives the position and maximum bending moment at any part of the cross-section of the two central strips of the slab.

When the span of the slab is small in relation to the thickness of the slab, it will be seen that the central strip of the slab will get a certain amount of support by the sides of the slab parallel to the central strip.

It will, of course, be obvious that the strips next to the side and parallel to the side cannot possibly have the deflection which will occur on the central strip.

As the ratio of the span to the depth of the slab increases, the effect of the supports which are parallel to the central strip decreases.

The tendency is to make slabs larger and larger, and a slab 9 inches thick may measure 30 feet by 60 feet.

Let us take a strip across the centre of the 60-foot edge having a span of 30 feet ; it is submitted in this case that the supports along the 30-foot edge will not do much to decrease the deflection of the central strip, which is 30 feet away from the shorter supporting edges. It is therefore urged that the use of the Grashof-Rankine formula for the bending moment of central strips is justifiable and reasonable.

It should be noted that the Grashof-Rankine rule purports to deal with the point of maximum stress in a homogeneous material, such as cast iron or plain concrete.

In the case of reinforced concrete, however, we are able to vary the spacing of the tensile reinforcement according to the bending moment at any part of the cross-section.

Narrow strips which are parallel to the sides and are near the sides are sustained for their whole length. They cannot deflect, and cannot have a camber in the direction of their length so long as the side supports remain rigid.

If we take a cross-section through the centre of the slab, we will find that the deflection at the midspan of the central strip will be a maximum, and the deflection at the midspan of the two outermost strips will be reduced to zero.

Moreover, the maximum bending moment will occur at some section of the central strip, while the bending moment on the cross-section of the two outermost strips (which are supported along one edge for the whole of their length) will be reduced and will approach zero.

Any exact calculation of the variation of the bending moment in a cross-section of all the adjacent strips will be nullified by any deflection of the side supports.

If we assume that the bending moments in a cross-section of all the adjacent strips will vary as the ordinates of a parabola, the average bending moment over the whole of the cross-section of the strips will be two-thirds of the maximum bending moment (on the central strip).

On this assumption it will be found that in the case of the square slab, at any rate, the Grashof-Rankine rule and the French rule give the same results. This may be shown as follows :

The bending moment reduction factor (for the shorter span) in the Grashof-Rankine rule

$$= \frac{l^4}{(l^4 + b^4)}.$$

Dividing the numerator and the denominator by l^4 will give

$$\frac{1}{1 + \frac{b^4}{l^4}} = \frac{1}{1 + \left(\frac{b}{l}\right)^4}.$$

In the case of the square slab :

$$\frac{1}{1 + \left(\frac{b}{l}\right)^4} = \tfrac{1}{2}$$

and the bending moment at the centre of the square slab

$$= \frac{Wb}{24} \times \frac{1}{2} = \frac{Wl}{24} \times \frac{1}{2} = \frac{Wl}{48}.$$

If the bending moments across the section of the slab are taken as varying as the ordinates of a parabola, the average bending moment across the slab will be two-thirds of the maximum bending moment, viz. the average bending moment across the whole of the given cross-section will be

$$\frac{2}{3} \times \frac{Wl}{48} = \frac{Wl}{72}.$$

This is, of course, the result which is given by the French rule.

The advantage of the Grashof-Rankine rule is, however, that it provides for the maximum bending moment at the centre of a slab, whereas the French rule seems more applicable to the average bending moment across the width of the slab without any consideration as to the manner and distribution of the bending moment and stresses, and without any regard to the fact that the maximum bending moment at the central strip of the section is probably one and a-half times the average bending moment over the whole width of the cross-section.

It might, therefore, be deemed desirable to consider the advisability of retaining unchanged the Grashof-Rankine rule for the maximum bending moment for the central strips of slabs, and taking a decreased bending moment for the parallel strips between the central strip and the sides ; the bending moment for the side strips being reduced according to some such law as here suggested.

This procedure is in strict accordance with the letter and the spirit of the Grashof-Rankine rule.

As a precautionary measure, it might furthermore be deemed advisable not to reduce the bending moment of the side strips to zero, but to adopt some working rule, which could, in a way, make some allowance for the fact that, if the side supports are the ribs of Tee-Beams, they must themselves suffer some deflection under load.

The tensile reinforcement parallel to and near the sides of the slabs will, by their intersection in the corners of the slabs, tend to reduce the extent of those diagonal tensions which must inevitably occur in the extreme corners of rectangular slabs when the central strips assume their permissible and intended deflection.

The varying bending moments across a section of the slab can be provided for, either by having uniform spacing of bars and varying the diameter of the bars ; or by varying the spacing and keeping the diameter of the bars uniform throughout the full width of the cross-section of the slab.

It will be found much more convenient, both in the design and in the actual execution of the work, to keep the diameter of the bars uniform and to vary the spacing.

APPENDIX IV.

STRENGTH OF PILLARS.

By E. F. Etchells.

The precise amount of the increase of strength in hooped pillars depends, *inter alia,* upon

1. The *form* of hooping (whether curvilinear or rectilinear, &c.).
2. The closeness of the *spacing* of the hoops in relation to the diameter of the hooped core.
3. The quantity of the hooping relative to the quantity of concrete in the core of the pillar.
4. The quality of the concrete.

Consequently the increase of strength may be shown to be proportional to the product of the four factors

$uf's'r$ where

$u =$ the *ultimate compressive* stress on concrete not hooped (in pounds per square inch).

$f' =$ a *form* coefficient which will vary according to whether the hooping is curvilinear or rectilinear, and which should, to a certain extent, represent the relative value of the form of hooping to resist deformation due to the expansion of the stressed core.

$s' =$ *spacing* ratio $= p/d$, the ratio of the pitch of the laterals to the diameter of the hooped core.

$V_h =$ *volume* of *helical* reinforcement in cubic inches.

$V =$ *volume* of hooped core in cubic inches.

$r = V_h/V =$ the *ratio* of volumes, *i.e.*, the *ratio* of the volume of *helical* or *horizontal* reinforcement to the volume of hooped core.

(Note.—If V is for the whole pillar, then V_h must be for the whole pillar. If V is for the volume of unit length of pillar, then V_h must be the volume of helical reinforcement in unit length of pillar.)

$p =$ the *pitch* or axial spacing of the laterals in inches.

$d =$ the *diameter* of the hooped core in inches.

By the use of suitable constants we may change a relationship of proportionality into a relationship of equality.

Let n_1, n_2, n_3, n_4 be such a series of numerical coefficients.

Thus instead of saying that "the increase of strength is *proportional* to the product of the four factors"

$$uf's'r$$

we may say that "the increase of strength is *equal to*"

$$(n_1u)(n_2f')(n_3s')(n_4r).$$

Let $n_1(n_2 f')(n_3 s')n_4 = m'$, then we may say that "the increase of strength due to the lateral hooping is equal to"

$$um'r.$$

The value of m' is fairly well known from experiments, and although each of its component factors has not yet been isolated, yet the separate effects of the "form" and "spacing" factor are traceable, so that instead of m' we can use the equation

$$m' = fs, \text{ where}$$

f and s are constants, depending, *inter alia,* upon the closeness of the spacing and the form of the laterals.

The *ultimate* compressive stress on a prism of concrete *not hooped* $= u$.

The working *compressive* stress on a *prism* of concrete (not hooped) $= c_p$.

The increase of strength due to hooping being

$$um'r$$

the total resistance of the hooped material will then be

$$= \left[\begin{matrix}\textit{Initial strength of non-}\\ \textit{hooped concrete}\end{matrix}\right] + \left[\begin{matrix}\textit{Increase of strength due to}\\ \textit{hooping of concrete}\end{matrix}\right]$$

$$= u + um'r$$

$$= u[1 + m'r].$$

Let $W_F =$ the *working factor* $= c_p/u =$ the reciprocal of the safety factor $= \frac{1}{S_F}$.

The safe compressive stress on the concrete within the hooped core $= c$ where

$$c = \frac{u[1 + m'r]}{S_F}$$

$$= W_F u[1 + m'r]$$

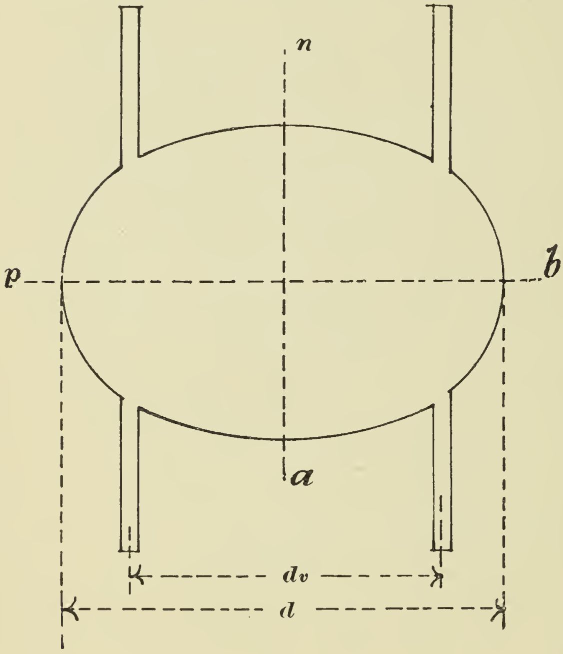
or since $m' = fs$

$$c = c_p[1 + fsr].$$

APPENDIX V.

THE MOMENT OF INERTIA OF SECTIONS OF REINFORCED CONCRETE.

By W. C. Unwin.



If m is the modular = ratio E_s/Ec of the coefficients of elasticity of steel and concrete, then an area A_t of steel is equivalent in resistance to mA_t of concrete. If A is the area of a section (including the area of reinforcing bars), and A_v the area of the vertical reinforcing bars, then the section is equivalent to a section of area $A = A_E + (m - 1)A_v$ of concrete only. This will be called the equivalent section.

The moment of inertia of a section about its neutral axis can always be put in the form

$$I = NAd^2,$$

where d is the depth at right angles to the neutral axis and N is a numerical coefficient depending on the form of the section. Thus for a rectangular section $I = \frac{1}{12}Ad^2$, and for a circular section $I = \frac{1}{16}Ad^2$.

In dealing with reinforced sections it is convenient in many cases to express the moment of inertia in terms of the equivalent area. The equivalent area is found by adding to the actual area of the section portions of a total area $(m - 1)\ A_v$ at the same distance from the neutral axis as the reinforcing bars.

The figure shows a section for which $p\ b$ is the plane of bending, and $n\ a$ the neutral axis passing through the centre of gravity of the section. The reinforcing bars are supposed symmetrical to the neutral axis. The projecting parts of total area $(m-1)A_v$ are the concrete areas equivalent to the steel. If I_c is the moment of inertia of the section without reinforcing bars, the moment of inertia with reinforcing bars is

$$I = I_c + \tfrac{1}{4}(m - 1)A_V d_v^2.$$

Thus for a rectangular section

$$I = \tfrac{1}{12}Ad^2 + \tfrac{1}{4}(m - 1)A_V d_v^2,$$

and the modulus of the section is

$$S_m = \tfrac{1}{6}Ad + \tfrac{1}{2}(m - 1)A_V \frac{d_v^2}{d}.$$

For a circular section

$$I = \tfrac{1}{16}Ad^2 + \tfrac{1}{4}(m - 1)A_V d_v^2,$$

and the section modulus is

$$S_m = \tfrac{1}{8}Ad + \tfrac{1}{2}(m - 1)A_V \frac{d_v^2}{d}.$$

EXAMPLE 1.—RECTANGULAR SECTION.

Let $m = 15$, $A_V = 0{\cdot}01A$, and $d_v = 0{\cdot}9d$. The equivalent area is $A_E = A + (m - 1)A_V = A + 14A_V$.

$$\begin{aligned} I &= \tfrac{1}{12}Ad^2 + \frac{14 \times 0{\cdot}81}{4 \times 100}Ad^2 \\ &= 0{\cdot}1117Ad^2. \\ \text{But } A &= 0{\cdot}877A_E \\ I &= 0{\cdot}098Ad^2 \\ S_m &= 0{\cdot}196Ad. \end{aligned}$$

In this case in the general expression $I = NAd^2$, $N = 0{\cdot}098$ when A_E is the equivalent section.

EXAMPLE 2.—CIRCULAR SECTION.

Let $m = 15$, $A_V = 0{\cdot}01A$, and $d_v = 0{\cdot}8d$. The area of the equivalent section is $1{\cdot}14A$, as before.

$$\begin{aligned} I &= \tfrac{1}{16}Ad^2 + \tfrac{1}{4} \times 14 \times {\cdot}01A \times {\cdot}64d^2 = 0{\cdot}0849Ad^2. \\ \text{But } A &= 0{\cdot}877A_E \\ I &= 0{\cdot}0745Ad^2 \\ S_m &= 0{\cdot}149Ad. \end{aligned}$$

EXAMPLE 3.—CIRCULAR SECTION WITH REINFORCING BARS ARRANGED IN A CIRCLE.

The reinforcing bars are nearly equivalent to a ring of steel of the same total area. Let $m = 15$, $A_V = 0{\cdot}01A$, and let the diameter of the circle of reinforcing bars be $d_v = 0{\cdot}8d$. The equivalent section is $A_E = 1{\cdot}14A$, as before.

$$\begin{aligned} I &= \tfrac{1}{16}Ad^2 + \frac{m-1}{8}A_V d_v^2 \\ &= 0{\cdot}0625Ad^2 + 1{\cdot}75 \times {\cdot}01A \times {\cdot}64d^2 \\ &= 0{\cdot}0737Ad^2 \\ &= 0{\cdot}0646Ad^2 \\ S &= 0{\cdot}1292Ad. \end{aligned}$$

It will be seen that if the value of N in the equation $I = NAd^2$ for simple circular and rectangular sections, and the reinforced sections are compared, the results are as follows:—

	Simple Sections	Reinforced Sections
Case I. . . .	$\frac{1}{12} = 0·0833$	0·098
Case II. . . .	$\frac{1}{16} = 0·0625$	0·0745
Case III. . . .	$\frac{1}{16} = 0·0625$	0·0646

The differences are not very great, so that while the value of N can always be found exactly when necessary for any proportion of reinforcement, there are cases such as that of columns where the value of N does not much affect the result, and where, from the nature of the calculation, great accuracy is impossible—for which a value of N can be assumed without any practically important error.

APPENDIX VI.

TENSILE AND SHEARING STRESSES IN WEB REINFORCEMENT.

By E. F. Etchells.

The purpose of this Appendix is to show that stresses on the web reinforcement can be deduced by comparing a reinforced concrete beam with a lattice girder.

In this comparison the compressive stresses in the web are assumed to be taken by the concrete and the tensile stresses are assumed to be taken up by the stirrups, whether such stirrups are vertical or inclined.

Take first the simplest case of a beam with vertical stirrups of pitch "p" equal to the lever arm "a."

This will give square "panels," and may be compared with a girder of the plain N type in which the diagonal compressive stresses are taken by the concrete, and the vertical tensile stresses are taken up by the stirrups.

An inspection of stress diagrams for this type of girder will show the nature and direction of the stresses we may expect.

We will find that the total tension on a given web member is equal to the total vertical shear at that part of the beam.

The resistance to this tensile force $= A_s . t$, where A_s is the *area* of the *stirrup* or the cross-sectional area of the two prongs of a U-shaped stirrup and t is the *tensile* stress on the stirrup.

(t should not be taken at more than 12,000 pounds per square inch.) Equating the external force to the internal resistance, we have $S = A_s t$.

Let us suppose that, instead of the web members being spaced at a distance apart equal to the lever arm a, they are spaced much closer.

Let N be the number of vertical stirrups in a horizontal distance a, and let p be the pitch of the stirrups centre to centre.

$$\text{Let } Np = a$$

$$\text{therefore } N = \frac{a}{p}.$$

Instead of using stirrups of large section with a pitch $= a$, we can use a number of stirrups of smaller sectional area so that the total area of stirrups in a horizontal distance equal to a will be NA_s. Since $NA_s = \frac{a\,A_s}{p}$ we are led to the equation

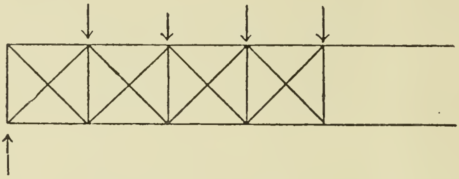
$$S = \frac{a\,A_s t}{p}, \text{ from which we obtain}$$

$A_s = \frac{p\,S}{a\,t}$, or, putting this into a form similar to that given on page 30, line 25, we get

$$A_s t = \frac{S\,p}{a} \text{ except } t \text{ is used instead of } s_s.$$

(As these two equations have been derived from such widely differing bases, they suggest the desirability of keeping the value of the tensile stress in stirrups down to the value of the shearing stress on such stirrups.)

In the case of a beam with inclined stirrups, the case is much more complicated, and the analogous lattice girder is of the form



In this case there are really four superimposed systems of bracing. The systems are of the form N and N inverted, and W and W inverted.

In two of the systems (the N systems) the stresses on the verticals neutralise each other.

If we assume that the loads are taken equally by each of the four systems, we will find that in two of the systems the diagonal tensions are taken up by the same inclined bar, so that we get only $\frac{2}{4}$ or $\frac{1}{2}$ of the total point or panel load taken up by any given inclined bar.

A further inspection of the stress diagrams for this form of latticing will show that the total tension on the diagonal will be equal to $\sqrt{2}$ times the total vertical shear at a vertical cross-section through the bar.

We are, therefore, led to the equation that the total tension on any given diagonal $= \frac{\sqrt{2}\,S}{2}$.

The resistance of the inclined stirrups with a horizontal pitch $= a$ will be $A_s t$ as before.

If, however, there are N stirrups in a horizontal distance a, bearing in mind that $N = \frac{a}{p}$, it is seen that the total resistance as before will be $\frac{a\,A_s t}{p}$.

We therefore obtain the full equation

$$\frac{\sqrt{2}\,S}{2} = \frac{a\,A_s t}{p}$$

$$\text{but } \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}} \text{ therefore}$$

$$\frac{S}{\sqrt{2}} = \frac{a\,A_s t}{p}$$

from which we obtain

$$A_s = \frac{S\,p}{a\sqrt{2}\,t}.$$

Converting this to the form given on page 31, line 3, we obtain

$$A_s t = \frac{S\,p}{a\sqrt{2}},$$

the only difference being that t is used instead of s_s.

It is urged that the use of t is preferable inasmuch as it reveals the principal function of web reinforcement, viz. the prevention of rupture by excessive *tensile* stresses in the web.

It will be seen, however, that the same results are obtained whether we start from the $\frac{dB}{dx}$ equation or from the lattice-girder analogies.

Many different assumptions may be made as to the method of loading and as to the form of lattice girder which would be equivalent to a given system of stirrups, but the examples given have been selected for purposes of comparison with the results obtained from the differential equation aforementioned.

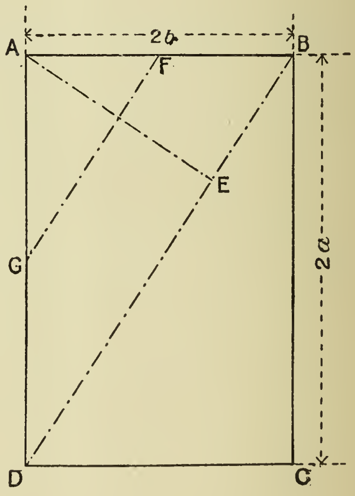
APPENDIX VII.

BACH'S THEORY OF THE RESISTANCE OF FLAT SLABS SUPPORTED ON ALL EDGES AND UNIFORMLY LOADED.

By W. C. Unwin.

The experiments of Professor Bach show that a flat square slab supported all round fractures along a diagonal, and the greatest stress is therefore on the diagonal section. It is the same apparently with rectangular slabs, though the evidence is not quite so clear. But if a diagonal fracture is assumed a very simple theory gives the stress.

Let the figure represent a rectangular slab with sides equal to $2a$ and $2b$ in inches. Let the diagonal BD $= d$; and the thickness of the slab $= h$ in inches. Draw AE perpendicular to BD and let AE $= c$ in inches. Draw FG bisecting the sides, then GF bisects AE. Let W be the total load on the slab in lbs.



The forces acting on the left of the diagonal section BD are the weight $\frac{1}{2}$ W of the half slab acting at $\frac{1}{3}\,c$ from BD and the supporting forces acting at G and F, which have a resultant $\frac{1}{2}$ W acting at $\frac{1}{2}\,c$ from BD.

Since
$$cd = 4ab,$$
$$d = 2\sqrt{(a^2+b^2)}$$
$$c = \frac{2ab}{\sqrt{(a^2+b^2)}}.$$

The bending moment on the diagonal section BD is

$$= \frac{W}{2}\left(\frac{c}{2}-\frac{c}{3}\right) = \frac{Wc}{12} = \frac{Wab}{6\sqrt{(a^2+b^2)}}.$$

The intensity of tensile or compressive stress is

$$f = \frac{6M}{dh^2} = \frac{Wab}{2h^2(a^2+b^2)}.$$

This may be put in the form

$$f = \frac{W}{h^2} \quad \frac{\frac{a}{b}}{\left(\frac{a^2}{b^2}+1\right)}.$$

$\frac{a}{b}=$	$f=$
1	$0{\cdot}25\,\frac{W}{h^2}$
1·5	$0{\cdot}23\,\frac{W}{h^2}$
2	$0{\cdot}20\,\frac{W}{h^2}$.

It would seem that if Bach's formula is to be used in calculating slabs, the reinforcing rods should be perpendicular to the diagonals of the rectangle.

APPENDIX VIII.

COMPARISON OF THE RESULTS GIVEN BY VARIOUS RULES FOR THE STRENGTH OF FLAT RECTANGULAR SLABS SUPPORTED ON ALL EDGES AND UNIFORMLY LOADED.

By William Dunn.

The theories of Professor Grashof and of Professor Rankine assume that the maximum bending stress on the slab is at the centre, where there are two principal stresses on planes normal to each other, these planes coinciding with the major and minor axes of the slab.

The stress on the plane formed by the major axis of the slab (which is the greater of the two principal stresses) may be found in a simple manner as follows:—

Let the *length* of the slab = $\boldsymbol{l}$, and the *breadth* = $\boldsymbol{b}$ (where $\boldsymbol{l}$ is equal to or greater than $\boldsymbol{b}$).

Calculate the bending moment on the slab (disregarding the end supports) as a beam supported or fixed at the sides only, of a span $\boldsymbol{b}$ under the total load on the slab. Multiply this bending moment by the *factor* for the span $\boldsymbol{b}$ in the following table, to allow for the effect of the end supports. The result is the actual bending moment on the long axis of the slab.

When	Grashof's and Rankine's Rule		French Government Rule	
$\frac{l}{b}=$	$F_b=\frac{l^4}{l^4+b^4}=$	$F_l=\frac{b^4}{l^4+b^4}$	$F_b=\frac{1}{1+2\frac{b^4}{l^4}}$	$F_l=\frac{1}{1+2\frac{l^4}{b^4}}$
1·0	0·50	0·50	0·33	0·33
1·5	0·83	0·16	0·71	0·09
2·0	0·94	0·05	0·89	0·03

The stress on the section formed by the long axis of the slab is found in the usual way by equating this actual bending moment B to the resistance moment R of that section.

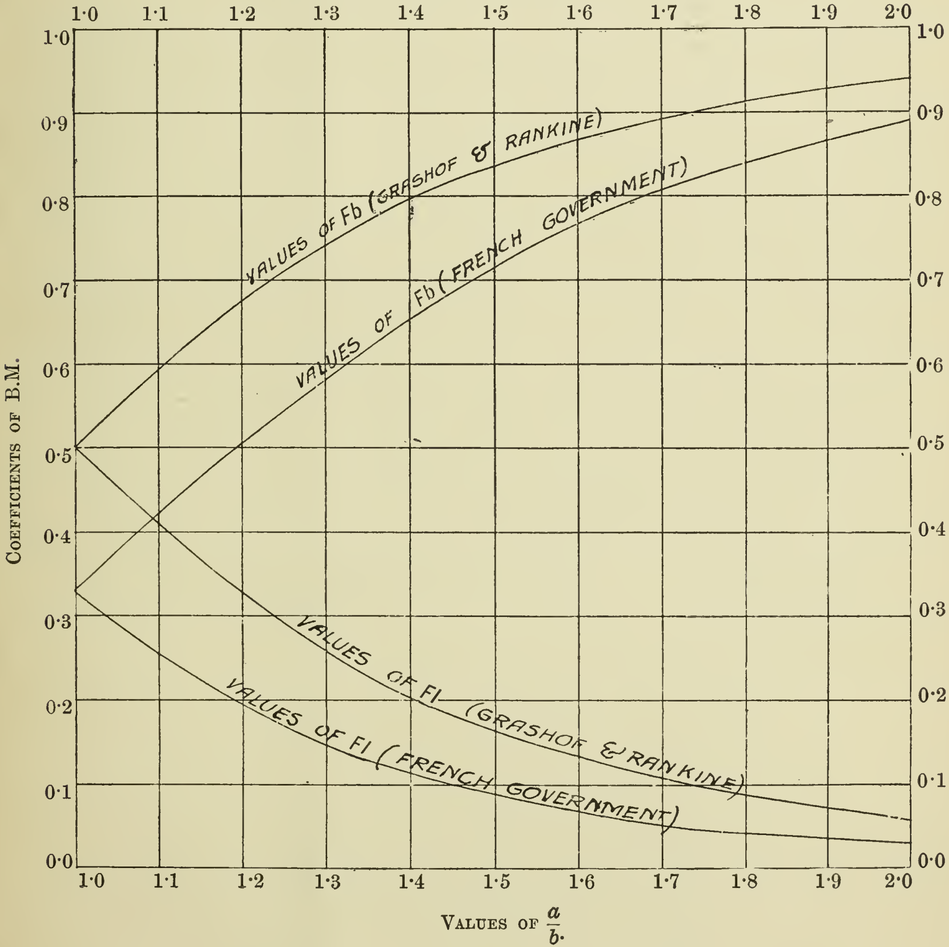
Similarly the stress on the plane formed by the minor axis of the slab is found by assuming the slab supported or fixed at the ends (disregarding the effect of the side supports), calculating the bending moment as if the slab were a beam of span l under the total load on the slab. Reduce the bending moment so found by the factor for the span $\boldsymbol{l}$ [F_l in the table above], and the result is the actual bending moment on the short axis of the slab.

The stress on the section formed by that axis is found as before by equating this B bending moment to the B resistance moment R of that section.

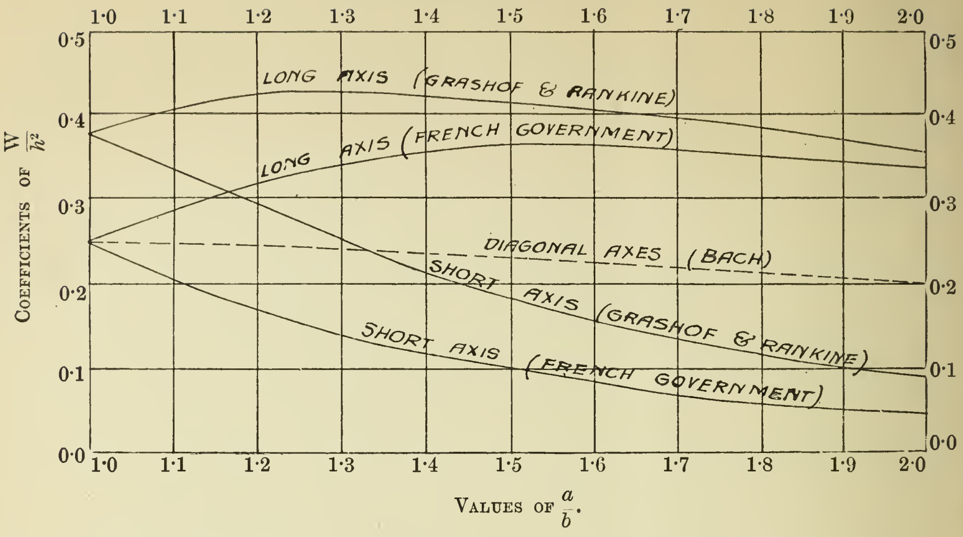
The reasoning by which we find the factors F_b and F_l is not entirely satisfactory, and other writers give other values. In the Instructions issued by the French Government to the Ingénieurs des Ponts et Chaussées with the Report of the Ministerial Commission du Ciment Armé the factors adopted give a greater importance to the effects of the third and fourth supports. The values of F_b and F_l, according to that report, are also given in the table above.

The maximum stresses on the sections as found by the foregoing rules when the slab is supported but not fixed all round are given in the table below, W being the total weight or load uniformly distributed over the slab d_s, the depth of the slab, and f the maximum stress due to bending.

BENDING MOMENTS (*Supported or Fixed*).



STRESSES (Supported only).



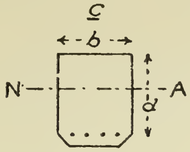
When $\frac{l}{b}=$	Values of f according to Grashof and Rankine		Values of f according to French Government Rule	
	On Long Axis	On Short Axis	On Long Axis	On Short Axis
1·0	$0·375\,\frac{W}{d_s^2}$	$0·375\,\frac{W}{d_s^2}$	$0·250\,\frac{W}{d_s^2}$	$0·250\,\frac{W}{d_s^2}$
1·5	$0·416\,\frac{W}{d_s^2}$	$0·183\,\frac{W}{d_s^2}$	$0·361\,\frac{W}{d_s^2}$	$0·101\,\frac{W}{d_s^2}$
2·0	$0·352\,\frac{W}{d_s^2}$	$0·088\,\frac{W}{d_s^2}$	$0·333\,\frac{W}{d_s^2}$	$0·045\,\frac{W}{d_s^2}$

These results may be more readily compared by the diagram given above.

It is implicitly assumed in the foregoing that the strength to resist bending is the same in both directions, so that the reinforcements longitudinal and transverse should be of equal area and at the same depth from the compressed face: they should be placed parallel to the ends and sides.

The stresses found by Bach's formula are also plotted on the diagram.

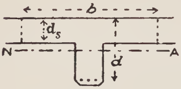
NS FOR SOLVING



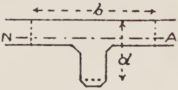
	(b) T beam (neutral axis within slab)	(c) Rectangular beam
	$\sqrt{m^2r^2+2mr}-mr$	
$2mr)$	$d\left(1-\frac{n_1}{3}\right)$	
$s_1)]$ $\left(1-\frac{s_1}{2n_1}\right)$	$\frac{c}{2}$	
rs_1+12mr $r)$	$\frac{cbd^2}{2}n_1\left(1-\frac{n_1}{3}\right)$	
s_1+12mr	$trbd^2\left(1-\frac{n_1}{3}\right)$	
s_1^2	$r=\frac{mc^2}{2t(mc+t)}$	
	rbd	

TABULAR STATEMENT OF THE EQUATIONS FOR SOLVING SINGLY REINFORCED BEAMS

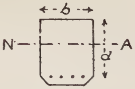
a



b



c



Description	(a) T beam (neutral axis outside slab)	(b) T beam (neutral axis within slab)	(c) Rectangular beam
Value of n_1	$\frac{s_1^2 + 2mr}{2(s_1 + mr)}$	$\sqrt{m^2r^2 + 2mr} - mr$	
Lever arm	$\frac{d(s_1^3 + 4mrs_1^2 - 12mrs_1 + 12mr)}{6mr(2 - s_1)}$ $= d\left[1 - \frac{s_1}{3}\left(\frac{3n_1 - 2s_1}{2n_1 - s_1}\right)\right]$	$d\left(1 - \frac{n_1}{3}\right)$	
Mean compression stress	$\frac{cmr(2 - s_1)}{s_1^2 + 2mr} = c\left(1 - \frac{s_1}{2n_1}\right)$	$\frac{c}{2}$	
Value of R_c	$cbdd_s \frac{s_1^3 + 4mrs_1^2 - 12mrs_1 + 12mr}{6(s_1^2 + 2mr)}$	$\frac{cbd^2}{2} n_1\left(1 - \frac{n_1}{3}\right)$	
Value of R_t	$tbd^2 \frac{s_1^3 + 4mrs_1^2 - 12mrs_1 + 12mr}{6m(2 - s_1)}$	$trbd^2\left(1 - \frac{n_1}{3}\right)$	
Values of r to give Stresses of c and t	$r = \frac{2mcs_1 - mcs_1^2 - ts_1^2}{2mt}$	$r = \frac{mc^2}{2t(mc + t)}$	
Value of r at which $n_1 = s_1$	$r = \frac{s_1^2}{2m(1 - s_1)}$		
Value of A_t	rbd	rbd	

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